

Non-perturbative renormalization and running of $\Delta F = 2$ four-fermion operators in the SF scheme

DIPARTIMENTO DI FISICA



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Motivations

Study of $\Delta F = 2$ transitions beyond the Standard Model (BSM). The most general $\Delta F = 2$ effective Hamiltonian is:

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \sum_{i=1}^5 C_i \mathcal{O}_i + \sum_{i=\{1,4,5\}} \tilde{C}_i \tilde{\mathcal{O}}_i$$

where $\mathcal{O}_i, \tilde{\mathcal{O}}_i$ are $\Delta F = 2$ four fermion operators:

$$\begin{aligned} \mathcal{O}_1 &= \bar{f} \gamma_\mu (1 - \gamma_5) q \bar{f} \gamma_\mu (1 - \gamma_5) q &= \mathcal{O}_{VV+AA} - \mathcal{O}_{VA+AV} \\ \mathcal{O}_2 &= \bar{f} \gamma_\mu (1 - \gamma_5) q \bar{f} \gamma_\mu (1 + \gamma_5) q &= \mathcal{O}_{VV-AA} + \mathcal{O}_{VA-AV} \\ \mathcal{O}_3 &= \bar{f} (1 + \gamma_5) q \bar{f} (1 - \gamma_5) q &= \mathcal{O}_{SS-PP} + \mathcal{O}_{PS-SP} \\ \mathcal{O}_4 &= \bar{f} (1 - \gamma_5) q \bar{f} (1 - \gamma_5) q &= \mathcal{O}_{SS+PP} - \mathcal{O}_{PS+SP} \\ \mathcal{O}_5 &= \bar{f} \sigma_{\mu\nu} (1 - \gamma_5) q \bar{f} \sigma_{\mu\nu} (1 - \gamma_5) q &= \mathcal{O}_{TT} - \mathcal{O}_{T\tilde{T}} \end{aligned}$$

and $\tilde{\mathcal{O}}_i$ are obtained from \mathcal{O}_i with $i \in \{1, 4, 5\}$ by exchanging $(1 - \gamma_5)$ with $(1 + \gamma_5)$.

For example

$$\langle \bar{F}_q^0 | \mathcal{O}_1 | F_q^0 \rangle \equiv \frac{8}{3} B_{F_q} f_{F_q}^2 m_{F_q}^2$$

defines the B -parameter for the mixing of the (neutral) mesons containing flavors f, q within the SM, while the matrix elements of $\mathcal{O}_2, \dots, \mathcal{O}_5$ appear only in BSM extensions.

Wilson like fermions break chirality \Rightarrow renormalization pattern of composite operators complicates with respect to the continuum (mixing with operators of different naïve chirality).

In the following Q_i and \mathcal{Q}_i are the parity-even (PE) and parity-odd (PO) part of \mathcal{O}_i and their mixing pattern reads [Donini *et al*, 1999]:

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix}_R = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \left[\mathbb{1} + \begin{pmatrix} 0 & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{21} & 0 & 0 & \Delta_{24} & \Delta_{25} \\ \Delta_{31} & 0 & 0 & \Delta_{34} & \Delta_{35} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & 0 & 0 \\ \Delta_{51} & \Delta_{52} & \Delta_{53} & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}_R = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1 \\ \mathcal{Q}_2 \\ \mathcal{Q}_3 \\ \mathcal{Q}_4 \\ \mathcal{Q}_5 \end{pmatrix}$$

⇒ The PO part thus mixes as in the case of a chirality-preserving regularisation.

Two possible strategies to avoid spurious mixing in the PE sector when using standard $O(a)$ -improved Wilson sea-quarks (e.g. CLS configurations):

- use twisted mass QCD at maximal twist for both flavours f and q (automatic $O(a)$ improvement but non-unitary setup) [Frezzotti & Rossi, 2004].
- use Ward Identities which relate the correlators of PE operators to those of PO ones with the insertion of a pseudo scalar density integrated over the whole space-time (unitary setup but no automatic $O(a)$ improvement) [Becirevic *et al.*, 2000]

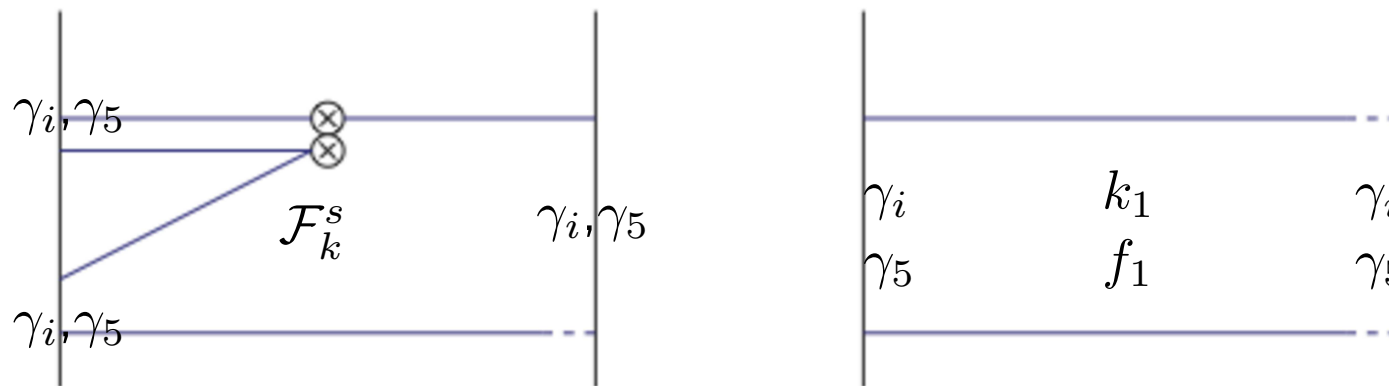
In both strategies we may restrict to the PO sector, which presents only the "physical" scale dependent mixing.

We have computed **non-perturbatively** in the **SF scheme** the renormalization matrix \mathcal{Z} at the hadronic matching scale $\mu_{\text{had}} = 1/L_{\text{max}}$ together with the **RG-running matrix** from the hadronic scale μ_{had} to the scale μ_{pt} where matching with perturbation theory at NLO can be safely made.

First time that the RG-running in presence of mixing has been computed **non-perturbatively** (thanks to the Schroedinger functional) over a range of scales μ which differ by 2 orders of magnitude $\mu \in [\Lambda_{QCD}, M_W]$.

In this exploratory study we have used non-perturbatively $O(a)$ -improved Wilson fermions with 2 massless sea flavors.

Non-perturbative renormalization in the SF scheme



$$\mathcal{A}_{k;\alpha}^s(L/2) = \frac{\mathcal{F}_k^s(L/2)}{f_1^{3/2-\alpha} k_1^\alpha} \quad \begin{pmatrix} \mathcal{Z}_{22} & \mathcal{Z}_{23} \\ \mathcal{Z}_{32} & \mathcal{Z}_{33} \end{pmatrix} \begin{pmatrix} \mathcal{A}_{2;\alpha}^{s_1} & \mathcal{A}_{2;\alpha}^{s_2} \\ \mathcal{A}_{3;\alpha}^{s_1} & \mathcal{A}_{3;\alpha}^{s_2} \end{pmatrix} = \begin{pmatrix} \mathcal{A}_{2;\alpha}^{s_1} & \mathcal{A}_{2;\alpha}^{s_2} \\ \mathcal{A}_{3;\alpha}^{s_1} & \mathcal{A}_{3;\alpha}^{s_2} \end{pmatrix}_{g_0^2=0}$$

and analogously for $k = \{4, 5\}$. The volume is L^4 and we have 5 possible sources s (pending on the combination of γ_5 and γ_i matrices on the boundaries) and three values of $\alpha \in \{0, 1, 3/2\}$.

Choosing one sub-set of operators (either $k = \{2, 3\}$ or $k = \{4, 5\}$), we combine the correlators with two different sources s_1 and s_2 to determine the four elements of the 2×2 sub-matrix of renormalization constants. We keep only those combinations for which the determinant of the matrix made with $\mathcal{A}_{k;\alpha}^s(L/2)$ is different from zero at tree level [S. Sint, 2001]. There are only 6 such combinations which, combined with the three different values of α give 18 different renormalization schemes.

The conditions above fix the renormalization matrix $\mathcal{Z}(g_0, a\mu)$ at the scale $\mu = 1/L$ for the 18 different schemes.

Step Scaling Functions (SSFs)

The starting point is the computation of the step scaling function (SSF) for the coupling and for the operators of the basis above:

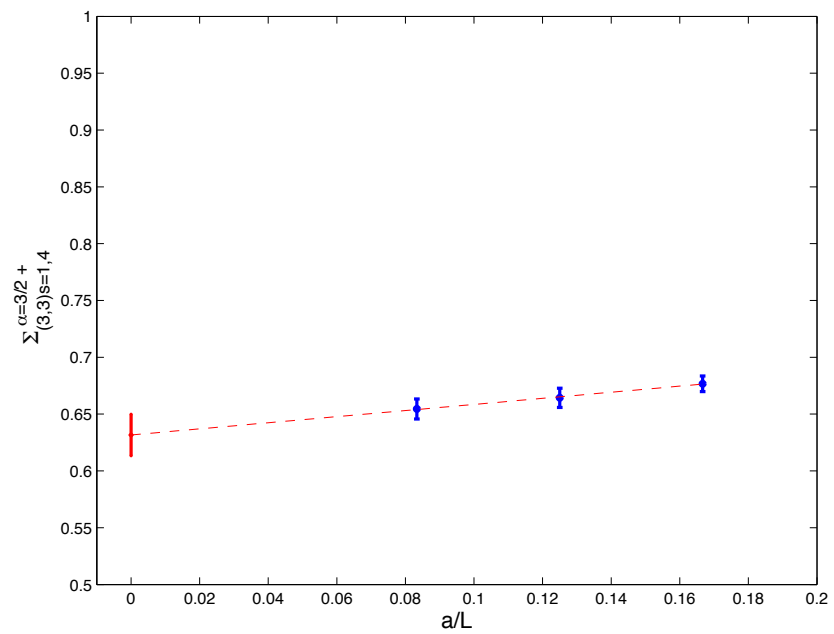
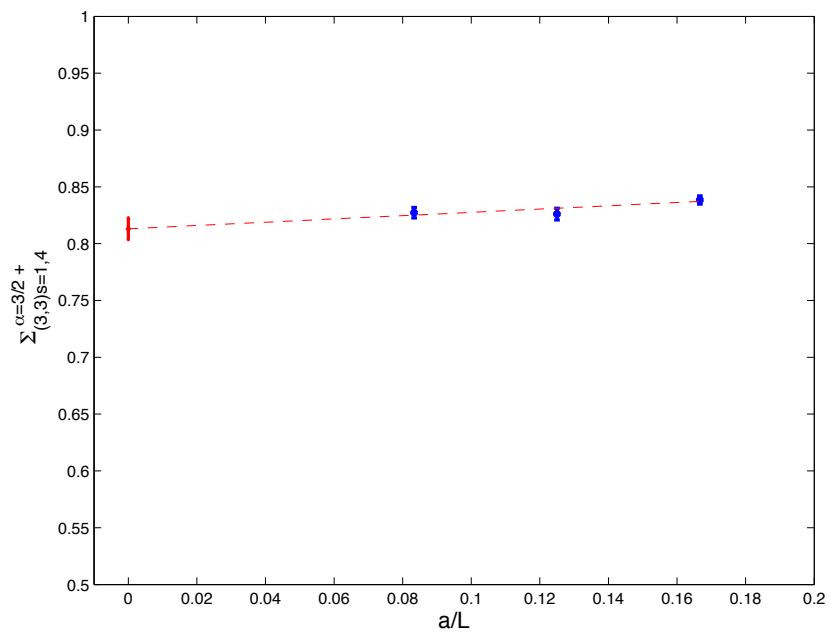
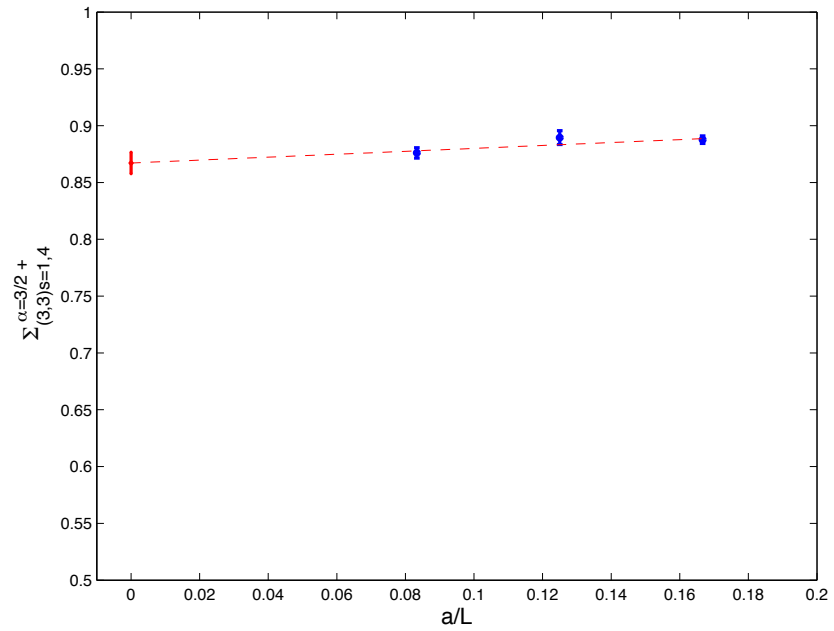
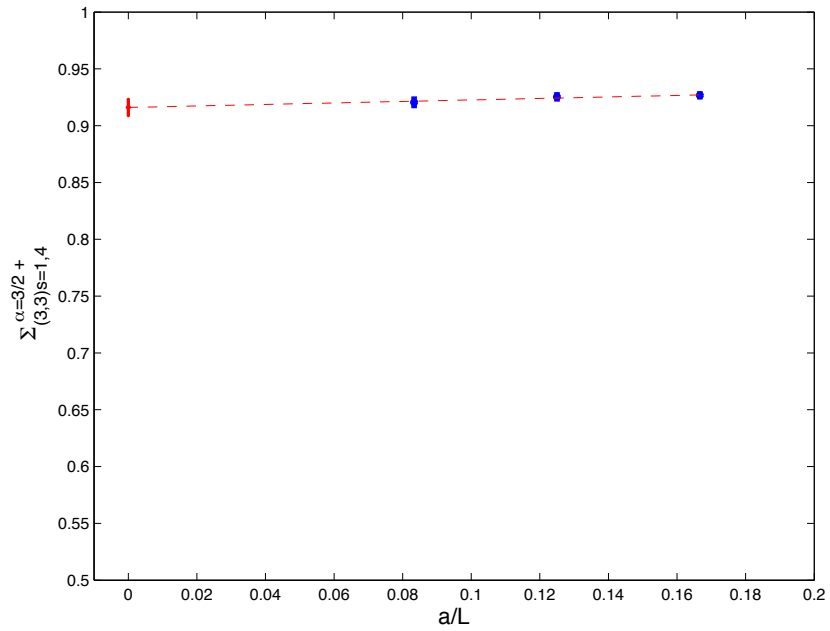
$$\Sigma_{\mathcal{Q}}(u, a/L) \equiv \mathcal{Z}(g_0, 2L/a) [\mathcal{Z}(g_0, L/a)]^{-1} \Big|_{u \equiv \bar{g}_{\text{SF}}^2(L)}^{m=m_{\text{cr}}}$$

$$\sigma_{\mathcal{Q}}(u) = U(\mu/2, \mu) \Big|_{\mu=1/L} = \lim_{a \rightarrow 0} \Sigma_{\mathcal{Q}}(u, a/L)$$

$$\sigma(u) \equiv \bar{g}_{\text{SF}}^2(2L) \quad u \equiv \bar{g}_{\text{SF}}^2(L)$$

The SSF $\sigma(u)$ has been computed in previous works by the Alpha coll. We have computed the matrix $\sigma_{\mathcal{Q}}(u)$ (for the 18 schemes) for 6 values of the coupling $u \in [0.9793, 3.3340]$.

Our operators are not $O(a)$ improved \Rightarrow continuum limit extrapolation linear in a/L . Performed on lattices with $L/a = \{6, 8, 12\}$ and $2L/a = \{12, 16, 24\}$ by tuning the β values at each L to obtain the chosen value of $u \equiv \bar{g}_{\text{SF}}^2(L)$.



Having the continuum limit $\sigma_Q(u)$ with $u \in [0.9793, 3.3340]$ we can perform a fit according to the perturbative guess $\sigma_Q(u) = \mathbf{1} + s_1 u + s_2 u^2 + s_3 u^3 + \dots$ where the s_i are 2×2 (sub-)matrices that can be related to the coefficients of the anomalous dimension matrix (ADM) and of the beta function

$$\gamma(g) = -g^2 \sum_{n=0}^{\infty} \gamma_n g^{2n} \quad \beta(g) = -g^3 \sum_{n=0}^{\infty} \beta_n g^{2n}$$

$$s_1 = \gamma_0 \ln 2 \quad s_2 = \gamma_1 \ln 2 + \beta_0 \gamma_0 (\ln 2)^2 + \frac{1}{2} \gamma_0^2 (\ln 2)^2$$

γ_1 is the NLO ADM in the SF scheme (we denote it by γ_1^{SF}). It can be obtained from the γ_1^{ref} already known in a reference scheme through the following two-loop matching relations:

$$\gamma_1^{\text{SF}} = \gamma_1^{\text{ref}} + [\chi_{\text{SF,ref}}^{(1)}, \gamma_0] + 2\beta_0 \chi_{\text{SF,ref}}^{(1)} + \beta_0^\lambda \lambda \frac{\partial}{\partial \lambda} \chi_{\text{SF,ref}}^{(1)} - \gamma^{(0)} \chi_g^{(1)}$$

$$\bar{g}_{\text{SF}}^2 = \chi_g(g_{\text{ref}}) g_{\text{ref}}^2 \quad (\mathcal{Q})_{\text{R}}^{\text{SF}} = \chi_{\text{SF,ref}}(g_{\text{ref}}) (\mathcal{Q})_{\text{R}}^{\text{ref}}$$

$$\chi(g) = 1 + \sum_{k=1}^{\infty} g^{2k} \chi^{(k)}$$

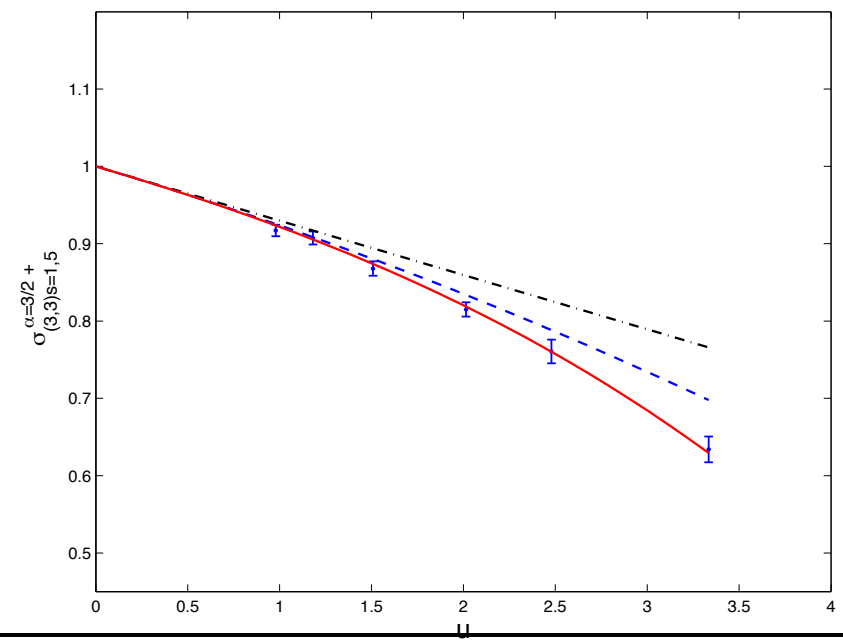
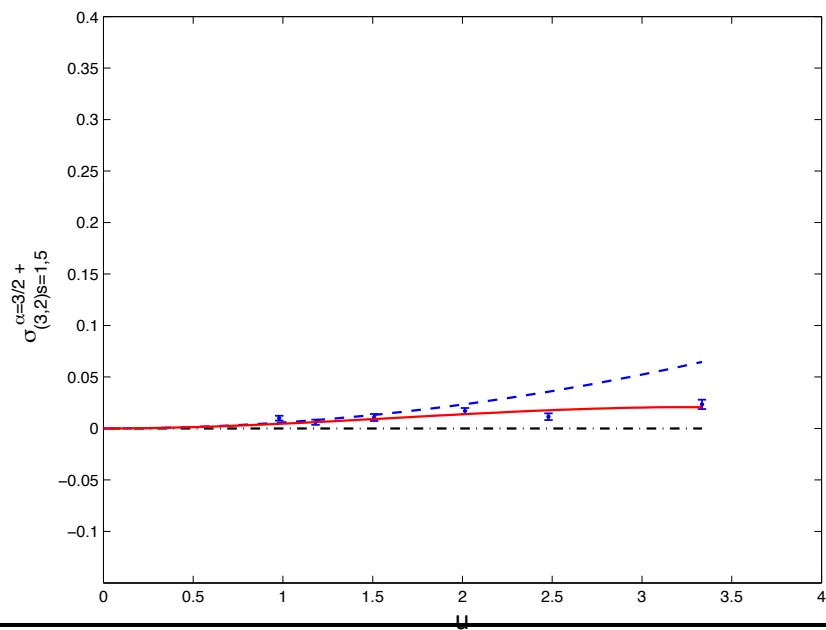
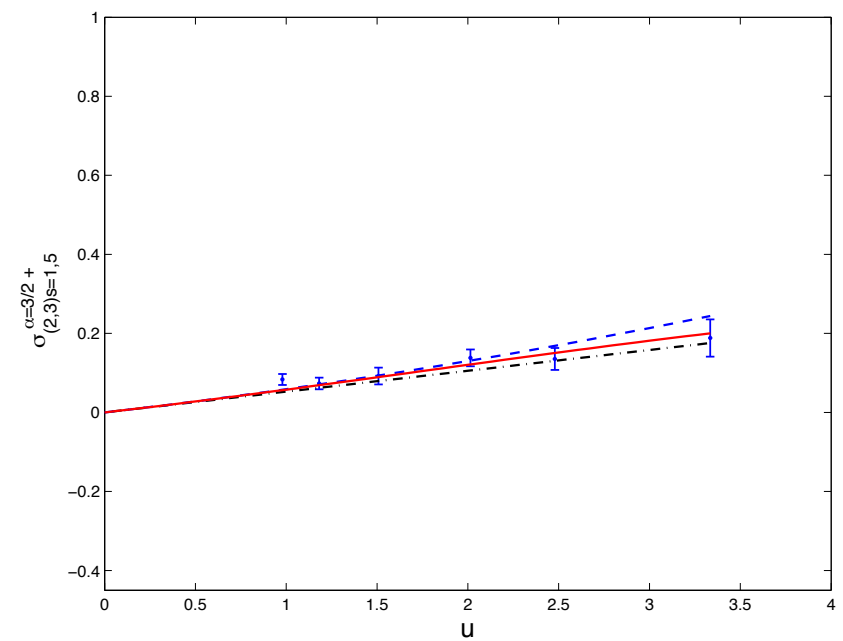
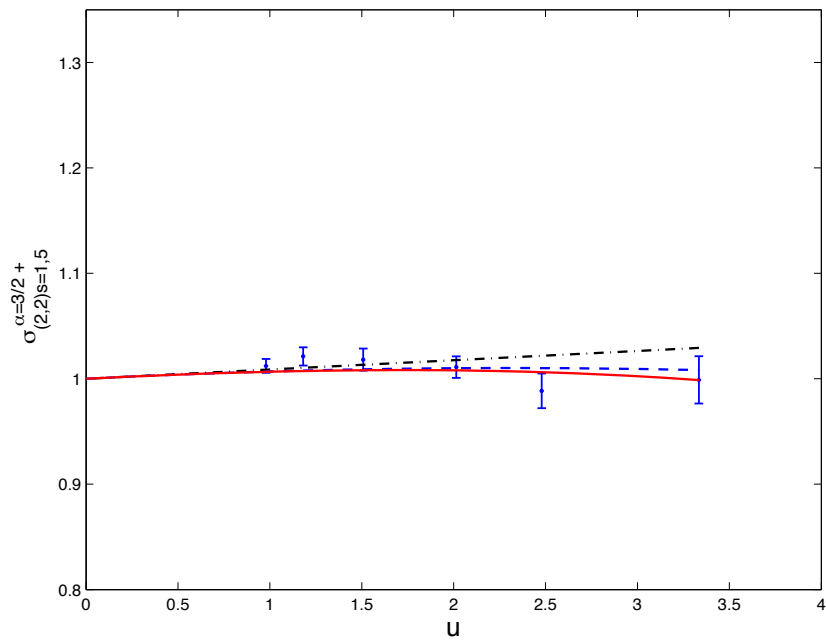
where λ is the gauge fixing parameter and β_λ is the beta function for the renormalised gauge fixing parameter $\lambda(\mu)$ (e.g. if we use as reference scheme the RI-MOM which depends on the gauge chosen. If we use $\overline{\text{MS}}$ there is no dependence upon the gauge).

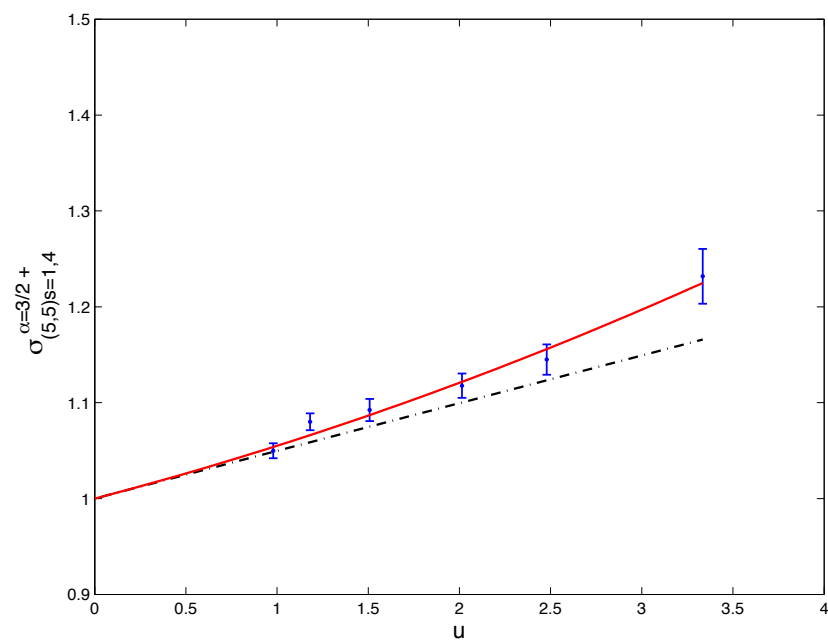
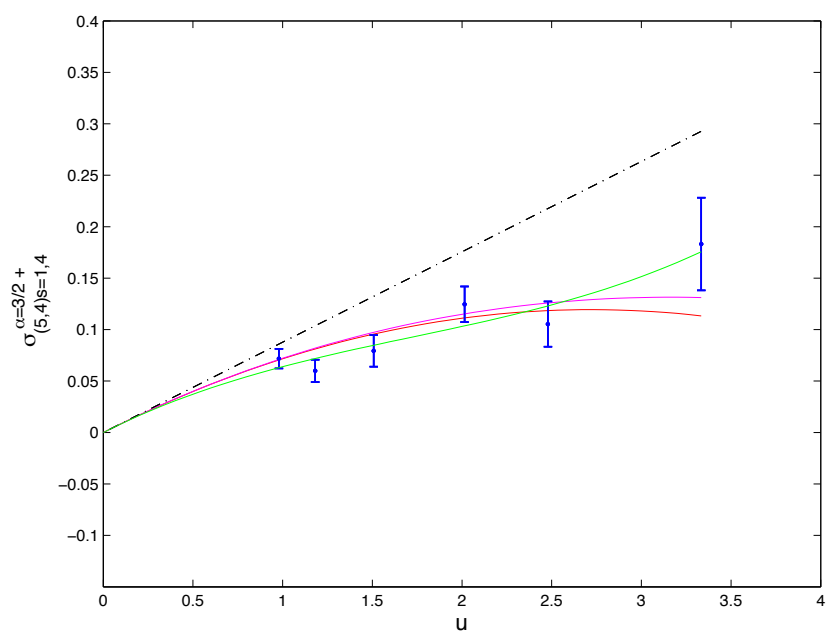
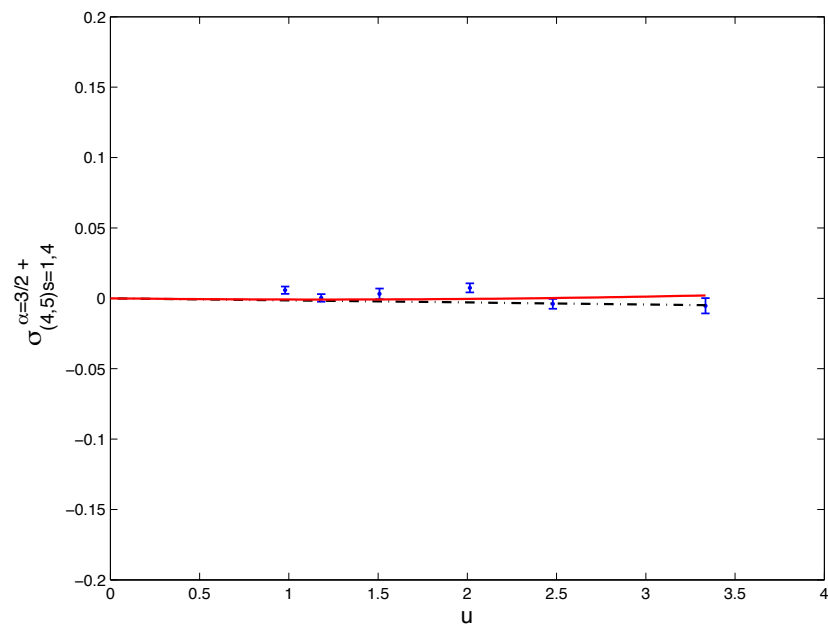
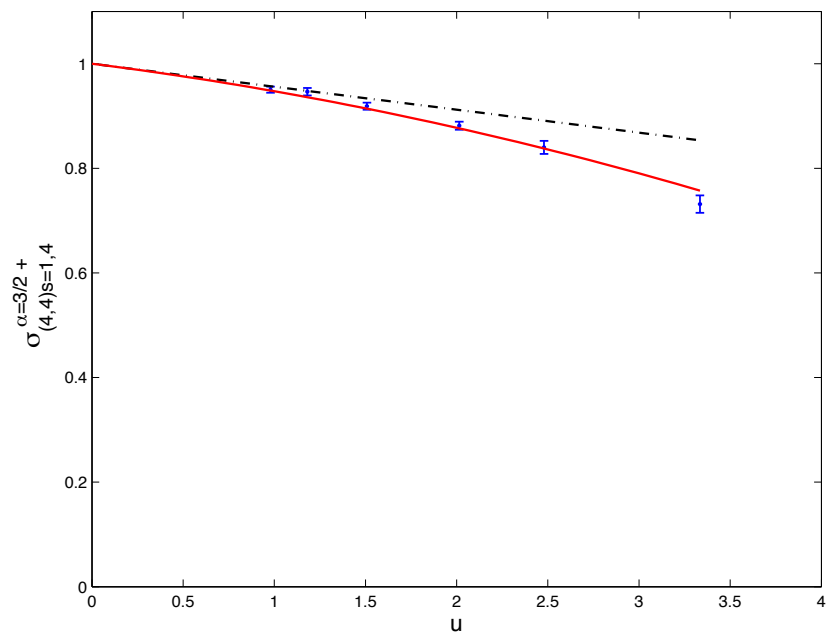
$\chi_{\text{SF,ref}}^{(1)} = \chi_{\text{SF,lat}}^{(1)} - \chi_{\text{ref,lat}}^{(1)}$ with $\chi_{\text{SF,lat}}^{(1)}$ and $\chi_{\text{ref,lat}}^{(1)}$ respectively the one-loop matching matrices between the bare lattice operator and either the SF or the reference scheme.

We have computed the matching matrix $\chi_{\text{SF,lat}}^{(1)}$ in perturbation theory at one-loop.

By using the results in literature for γ_1^{ref} [Buras *et al.*, 2000], $\chi_{\text{ref,lat}}^{(1)}$ [Gupta *et al.* 1998; Capitani *et al.*, 2001] and $\chi_g^{(1)}$ [Sint & Sommer, 1996] we have computed γ_1 in the SF scheme, for the moment only for the 2-3 sub-matrix (we are performing some checks on the 4-5 sub-matrix).

$\gamma_1^{\text{SF}} \Rightarrow$ compute s_1 and s_2 using the formula above and keep s_3 as a free parameter in the fit (2-3 matrix). In the case of 4-5 we have for the moment fixed only s_1 and kept s_2 as the only free parameter. With the exception of $(\sigma_Q)_{54}$ where we found the need to introduce s_3 as a second free parameter.





Non-perturbative renormalization group running

Once the SSF matrix has been fitted on the whole range of couplings $u \in [0.9793, 3.3340]$, the non-perturbative running can be obtained from the scale $\mu_{\text{had}} = 1/L_{\text{max}}$ to the scale $\mu_{\text{pt}} = 2^n \mu_{\text{had}}$ where n is the number of steps performed and where L_{max} is such that $\sigma^{-1}(\bar{g}^2(L_{\text{max}}))$ belongs to the upper end of the range of couplings simulated:

$$U(\mu_{\text{pt}}, \mu_{\text{had}}) = [\sigma_{\mathcal{Q}}(u_1) \dots \sigma_{\mathcal{Q}}(u_n)]^{-1}, \quad u_i = \bar{g}^2(2^i \mu_{\text{had}})$$

In the present case, with 7 steps we have $\mu_{\text{had}} \approx 0.44 \text{ GeV}$ while $\mu_{\text{pt}} \approx 56 \text{ GeV}$ where one can safely match with the perturbative RG-evolution at the NLO.

Matching with perturbation theory at μ_{pt}

If operators mix, the RG-evolution is formally obtained by using

$$U(\mu_2, \mu_1) = \text{T exp} \left\{ \int_{\bar{g}(\mu_1)}^{\bar{g}(\mu_2)} \frac{\gamma(g)}{\beta(g)} dg \right\}$$

A convenient form to use this formula at the NLO order is obtained by separating the LO part in the following way:

$$U(\mu_2, \mu_1)_{\text{LO}} = \left[\frac{\bar{g}^2(\mu_2)}{\bar{g}^2(\mu_1)} \right]^{\frac{\gamma_0}{2\beta_0}}$$

$$U(\mu_2, \mu_1) = [W(\mu_2)]^{-1} U(\mu_2, \mu_1)_{\text{LO}} W(\mu_1)$$

where $W(\mu)$ satisfies a new RG-equation and is regular in the UV: $\lim_{\mu \rightarrow \infty} W(\mu) = \mathbf{1}$

The RGI operators are easily defined using the above form:

$$Q^{\text{RGI}} \equiv \tilde{U}(\mu) (\mathcal{Q}(\mu))_{\text{R}} = \left[\frac{\bar{g}^2(\mu)}{4\pi} \right]^{-\frac{\gamma_0}{2\beta_0}} W(\mu) (\mathcal{Q}(\mu))_{\text{R}}$$

This formula is still valid non-perturbatively. One can use it to perform the matching at μ_{pt} with the NLO perturbative evolution:

$$Q^{\text{RGI}} = \left[\frac{\bar{g}^2(\mu)}{4\pi} \right]^{-\frac{\gamma_0}{2\beta_0}} W(\mu_{\text{pt}}) U(\mu_{\text{pt}}, \mu_{\text{had}}) (\mathcal{Q}(\mu_{\text{had}}))_{\text{R}}$$

where one can expand $W(\mu)$ in perturbation theory $W(\mu) \simeq \mathbf{1} + \bar{g}^2(\mu)J(\mu) + O(\bar{g}^4)$ where J depends on the ADM at the NLO γ_1 and satisfies:

$$\frac{\partial}{\partial \mu} J(\mu) = 0 \quad J - \left[\frac{\gamma_0}{2\beta_0}, J \right] = \frac{\beta_1}{2\beta_0^2} \gamma_0 - \frac{1}{2\beta_0} \gamma_1$$

We have solved the previous equations and obtained the matrix J in the SF scheme. The non-perturbative RGI evolution matrix $\tilde{U}(\mu)$ is plotted below against the LO and the NLO perturbative ones for the 2-3 operators.

The total RGI renormalization matrix is defined from $Q^{\text{RGI}} \equiv \mathcal{Z}^{\text{RGI}}(g_0) Q(g_0)$ where

$$\mathcal{Z}^{\text{RGI}}(g_0) = \tilde{U}(\mu_{\text{pt}}) U(\mu_{\text{pt}}, \mu_{\text{had}}) \mathcal{Z}(g_0, a\mu_{\text{had}})$$

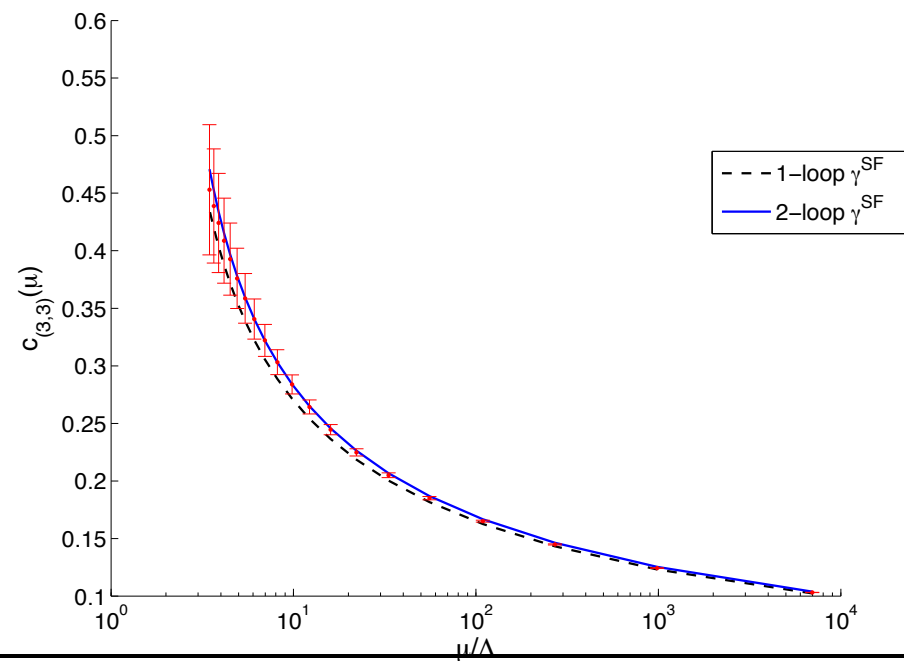
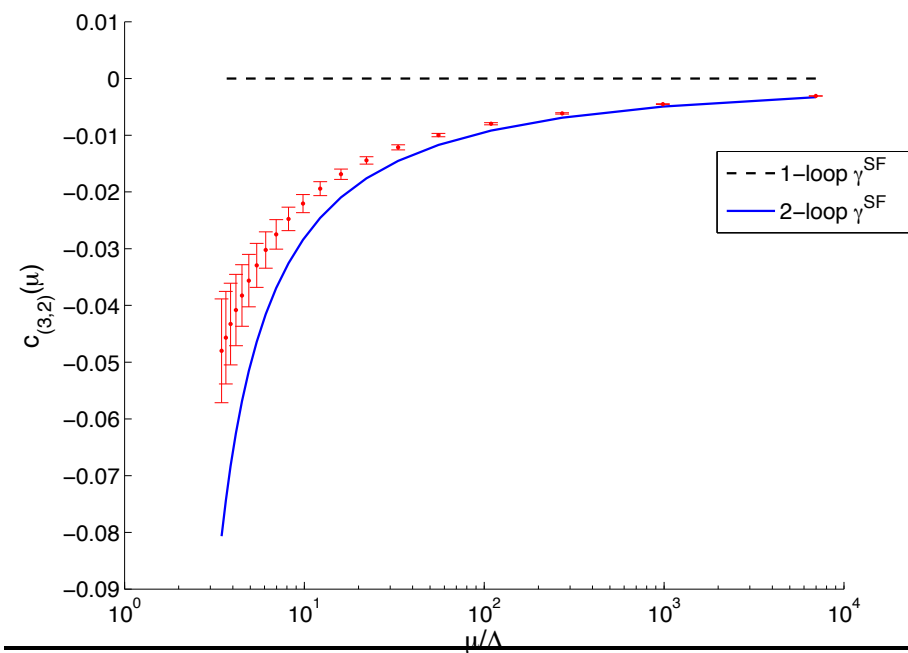
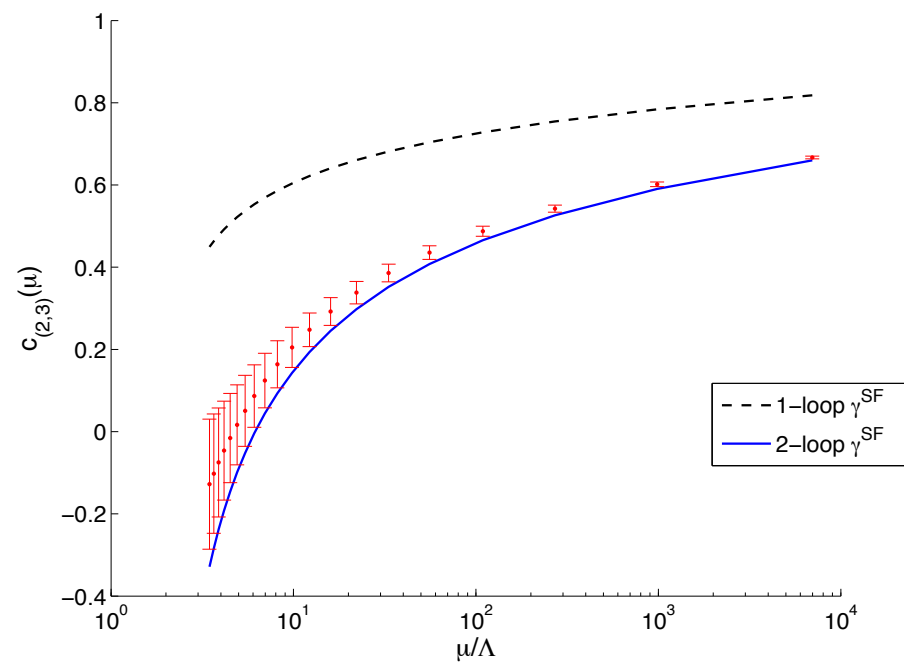
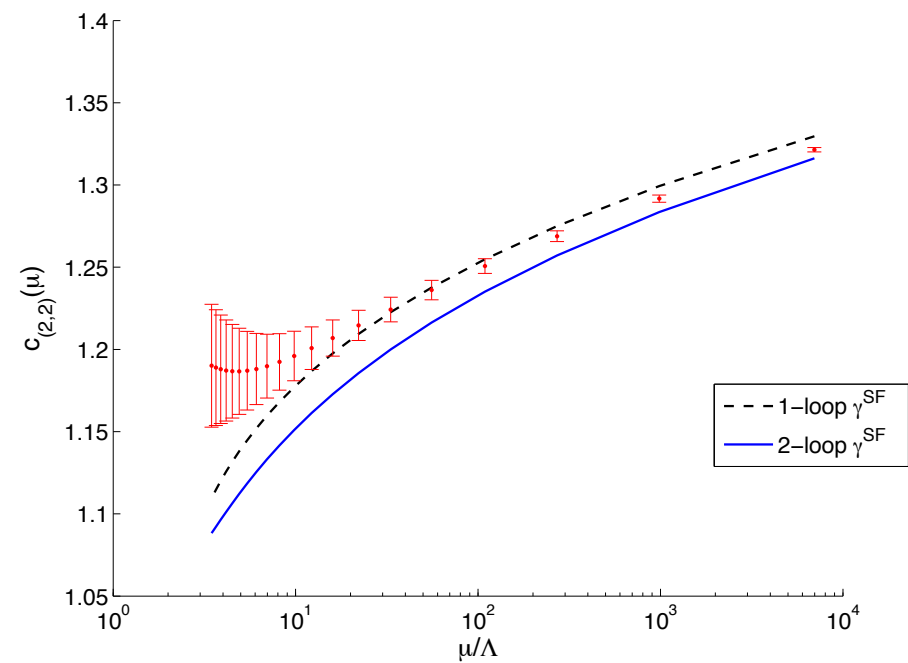
and $\mathcal{Z}(g_0, a\mu_{\text{had}})$ is the non-perturbative renormalization constant matrix computed at the hadronic scale.

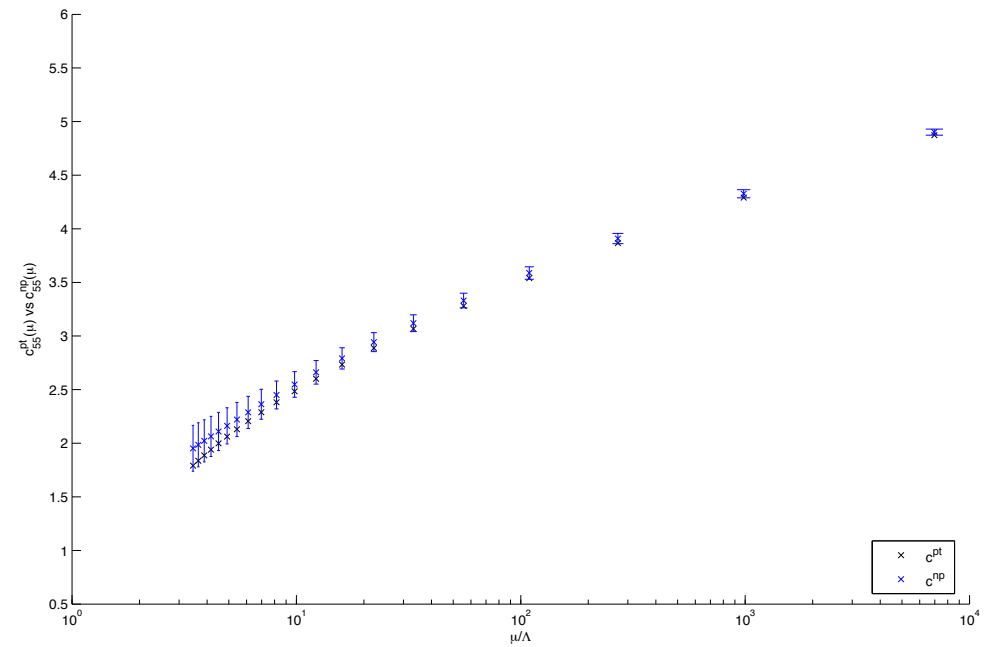
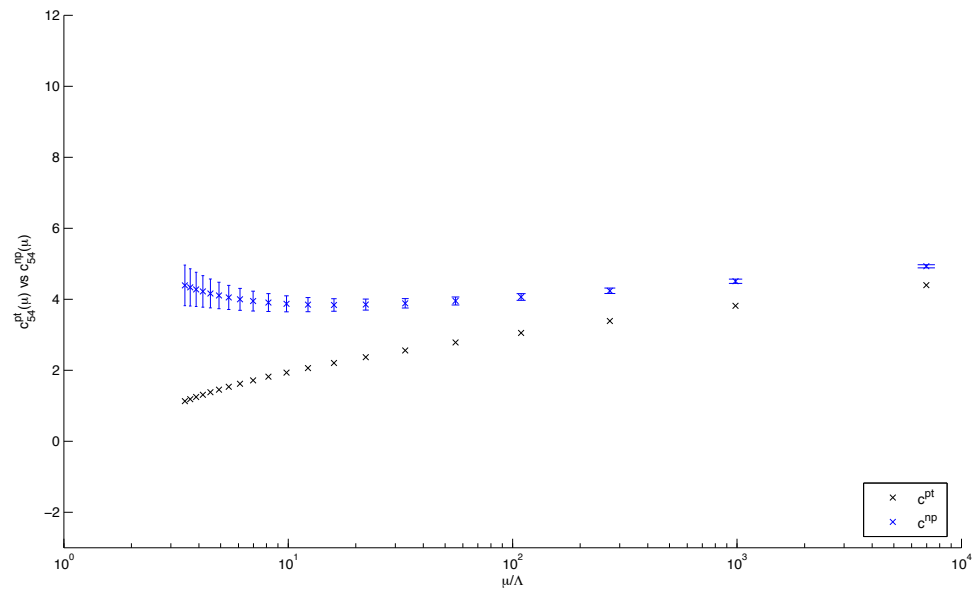
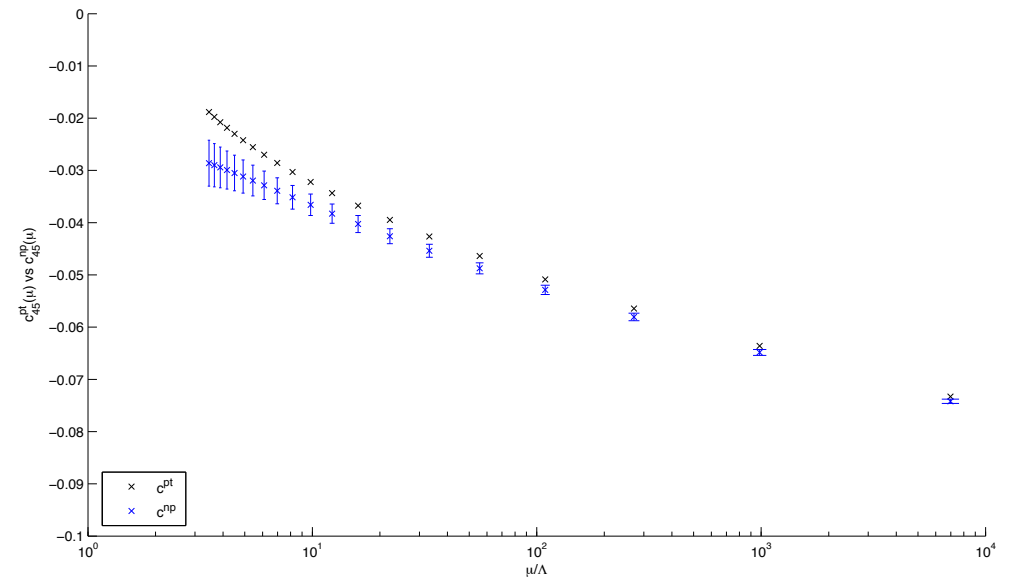
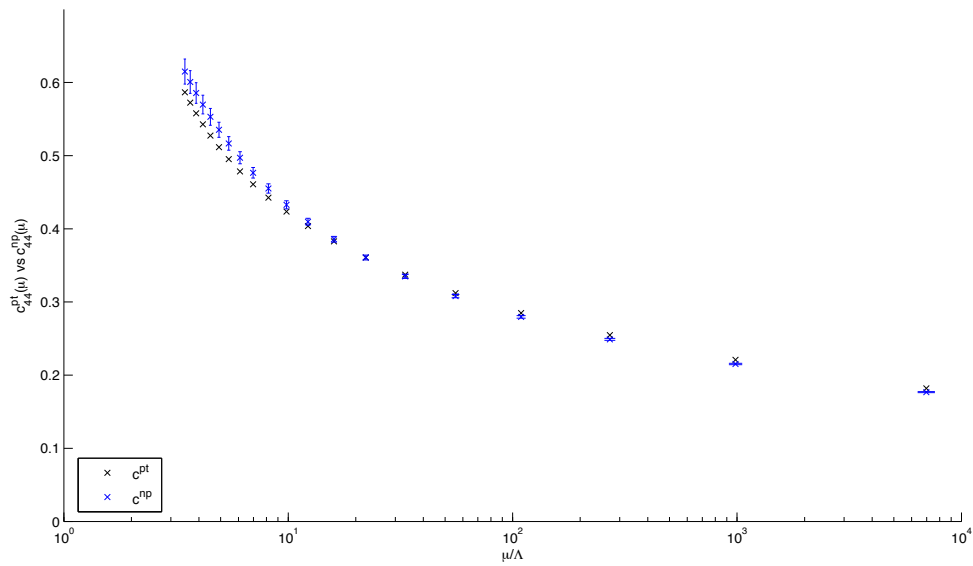
$\mathcal{Z}(g_0, L/a)$ has been computed at three values of $\beta \in \{5.20, 5.29, 5.40\}$ useful for large volume simulations, on three volumes for each β ($L/a = \{4, 6, 8\}$). By interpolating to L_{max} for which $\bar{g}^2(L_{\text{max}}) = 4.61$ one get $\mathcal{Z}(g_0, a\mu_{\text{had}})$ for each β .

Finally one can interpolate $\mathcal{Z}(g_0, a\mu_{\text{had}})$ in β using the formula

$$\mathcal{Z}^{\text{RGI}} = a_0 + a_1(\beta - 5.2) + a_2(\beta - 5.2)^2$$

Check: up to lattice artefacts $\mathcal{Z}^{\text{RGI}}(g_0)$ must be scale and scheme independent.





Conclusions and outlook

- general philosophy: dynamical Wilson fermions can be simulated efficiently and SF allows fully non-perturbative computation of the renormalization and running
⇒ use Ward Identities or tmQCD to solve intricate mixing problems for 4-fermion operators.
- Non-perturbative renormalisation important to determine hadronic matrix elements within a few percent accuracy. For the complete basis of 4-fermion operators, mixing it is particularly important.
- First exploratory non-perturbative computation of the RG-running in presence of mixing on a range of scales which varies over 2 orders of magnitudes thanks to the SF scheme.
- non-perturbative effects depend on the scheme but seems sizeable even at scales of 2-3 GeV.

- (regularization dependent) matching factors $\mathcal{Z}^{\text{RGI}}(g_0)$ obtained within a 5 – 10% accuracy.
- The same strategy is immediately portable to $N_f = 2 + 1$ dynamical simulations. The present data were not generated through a dedicated run. Increase of statistical accuracy easy to reach.
- By using χSF scheme [S.Sint, 2005-2011] (talks by Dalla Brida and Vilaseca) one has automatic $O(a)$ improvement and needs 3-point functions instead of 4-point functions \Rightarrow further increase in statistical accuracy.

